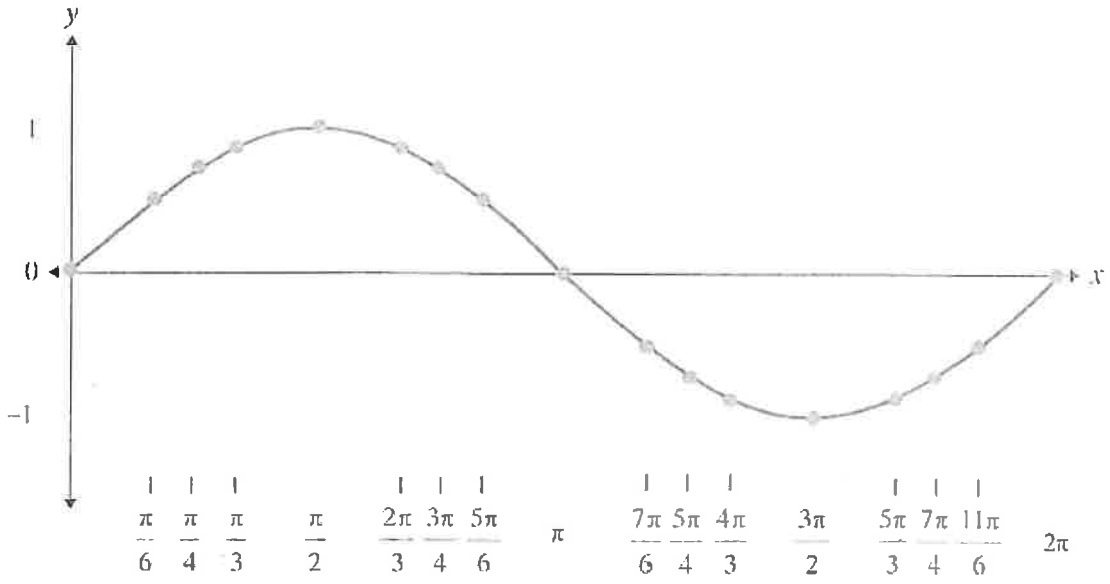
Sections 6.4 Part A
Graphs of the Sine and Cosine Functions

Here are many of the points that lie on the graph of the function $f(x) = sin\, x$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	
$sin\, x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	0	

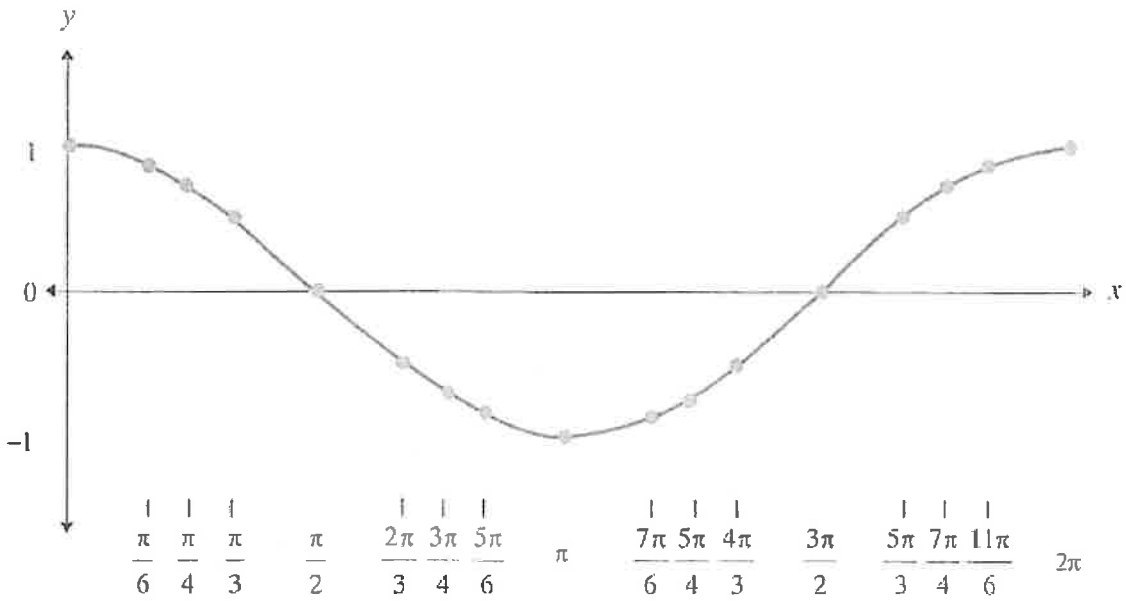
x	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π		
$sin\, x$	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0		



Here are many of the points that lie on the graph of the function $f(x) = cos\, x$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	
$cos\, x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	-1	

x	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π		
$cos\, x$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	1		



Properties of the Sine and Cosine Functions:

-Domain : $x = (-\infty, \infty)$

-Range : $-1 \leq cos\, x \leq 1$ $\quad\quad$ $-1 \leq sin\, x \leq 1$

$sin\, x$ is an odd function (symmetric about the origin)

$cos\, x$ is an even function (symmetric about the y-axis)

The sine and cosine functions are both periodic functions with a period of 2π

$$cos\, x = cos\,(x + 2\pi k)$$

$$sin\, x = sin\,(x + 2\pi k)$$ Where k is any integer.

Transformations of the Sine and Cosine Functions:

Given function $f(x) = A\,sin(Bx + C)$

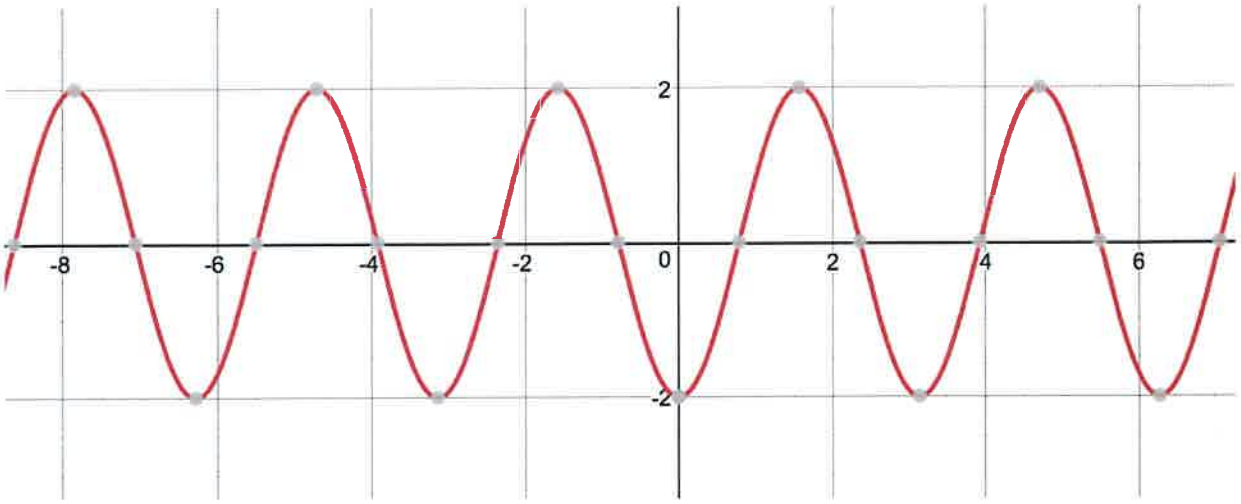
The **phase shift** is amount the interval is shifted to the left or right. The shift is to the left if the phase shift is negative and it is to the right if the phase shift is positive. Phase shift $= -\frac{C}{B}$

The **period** is the length of the interval required to complete one cycle. Period $= \frac{2\pi}{B}$

The **Amplitude** is the height of the wave and is given by:
Amplitude $= |A|$

https://www.desmos.com/calculator

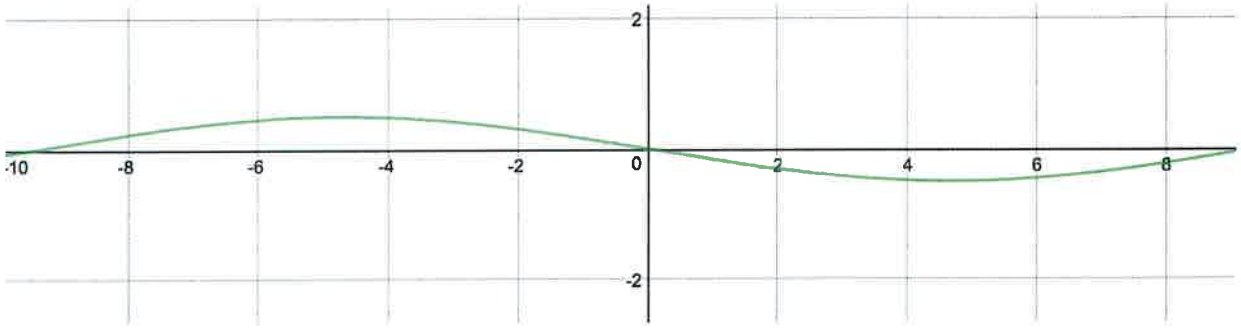
Example 1: $f(x) = 2\cos(2x + \pi)$ $A=2,\ B=2,\ C=\pi$



Amplitude $= 2$, Period $= \frac{2\pi}{B} = \pi$

Phase shift $= -\frac{C}{B} = \frac{-\pi}{2}$

Example 2: $g(x) = -\frac{1}{2}\sin\left(\frac{1}{3}x\right)$ $A = -\frac{1}{2},\ B = \frac{1}{3},\ C = 0$

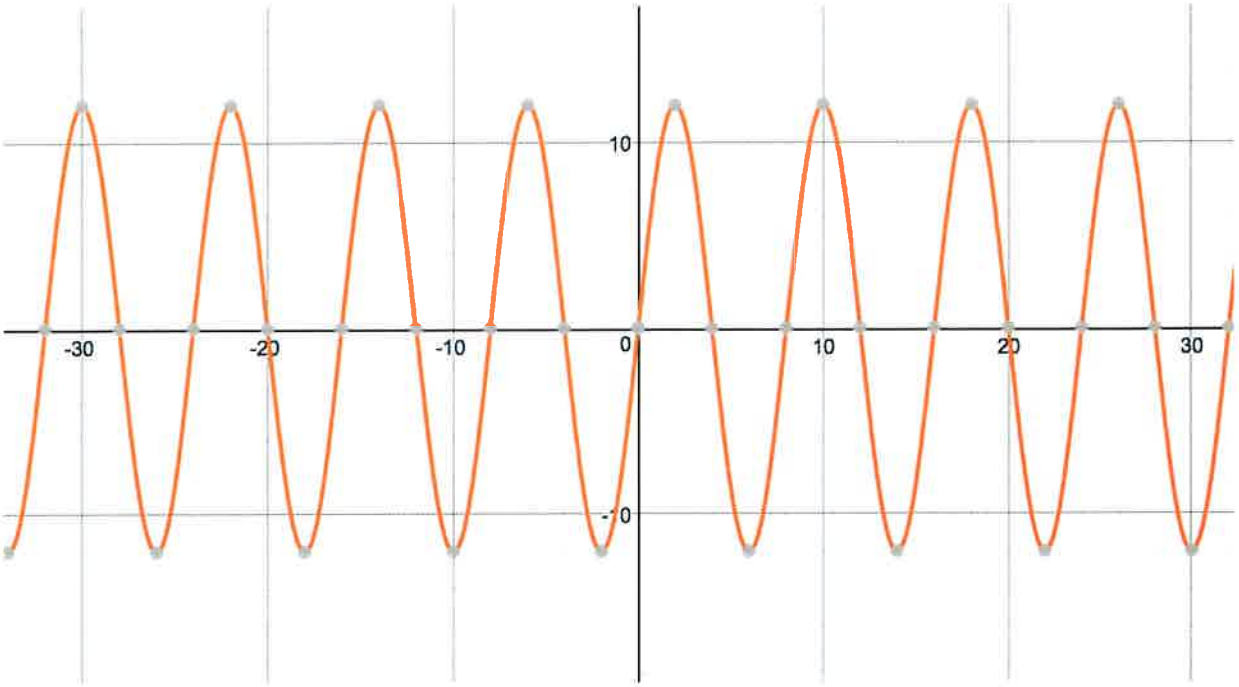


Amplitude $= \frac{1}{2}$, Period $= \frac{2\pi}{B} = \frac{2\pi}{\frac{1}{3}} = 6\pi$

Phase shift $= -\frac{C}{B} = 0$

Example 3: $h(x) = -12\cos\left(\frac{\pi}{4}x + \frac{\pi}{2}\right)$

$A = -12$, $B = \frac{\pi}{4}$, $C = \frac{\pi}{2}$



Amplitude = 12

Period = $\frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{4}} = 8$

Phase shift = $-\frac{C}{B} = \frac{-\frac{\pi}{2}}{\frac{\pi}{4}} = -2$